Heavy Flavor Physics

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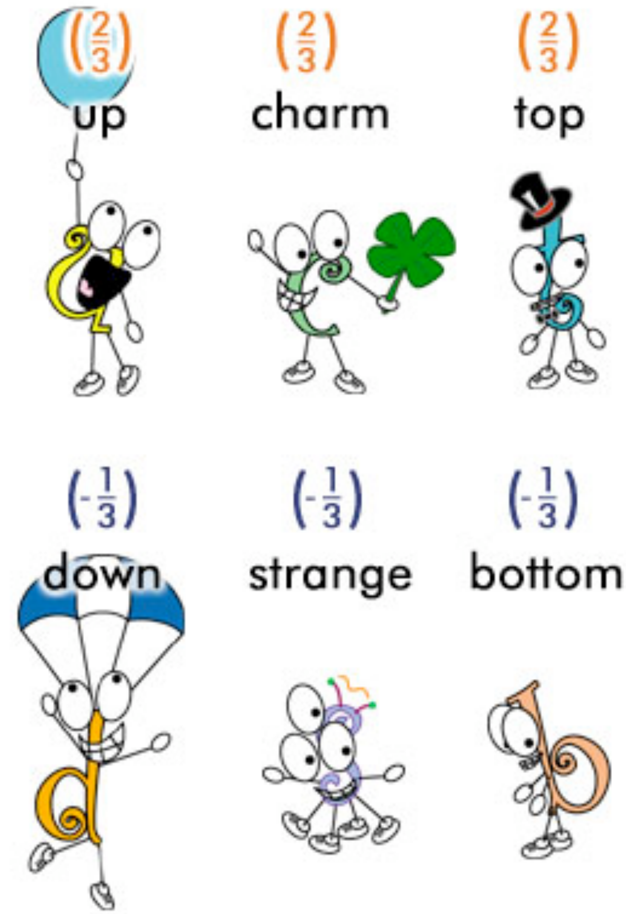
Outline

- Lecture 1:
  - Introduction and motivation
  - Yukawa couplings
  - CKM matrix, unitarity triangle
  - Effective weak Hamiltonian
- Lecture 2:
  - B-$\bar{B}$ mixing amplitude
  - Inclusive processes: OPE and applications ($B \to X_{c,u} l\nu$, $B \to X_s\gamma$)
  - Exclusive processes: trees and penguins, CP violation, searches for New Physics

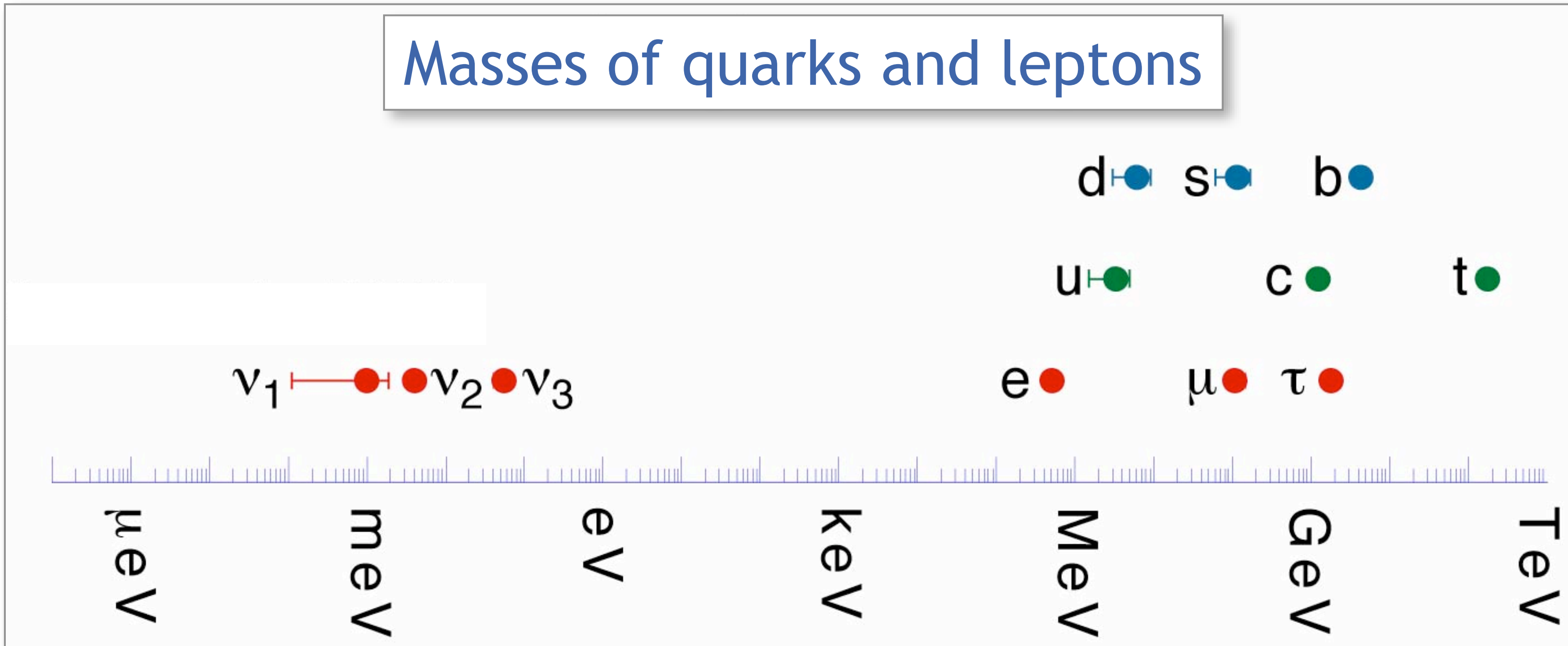
Flavor physics

- What is “flavor”?
- Generations: triplication of fermion spectrum without obvious necessity
- Dynamical explanation of flavor?
- Equally mysterious as dynamics of electroweak symmetry breaking
- Connection between two phenomena?



Flavor physics

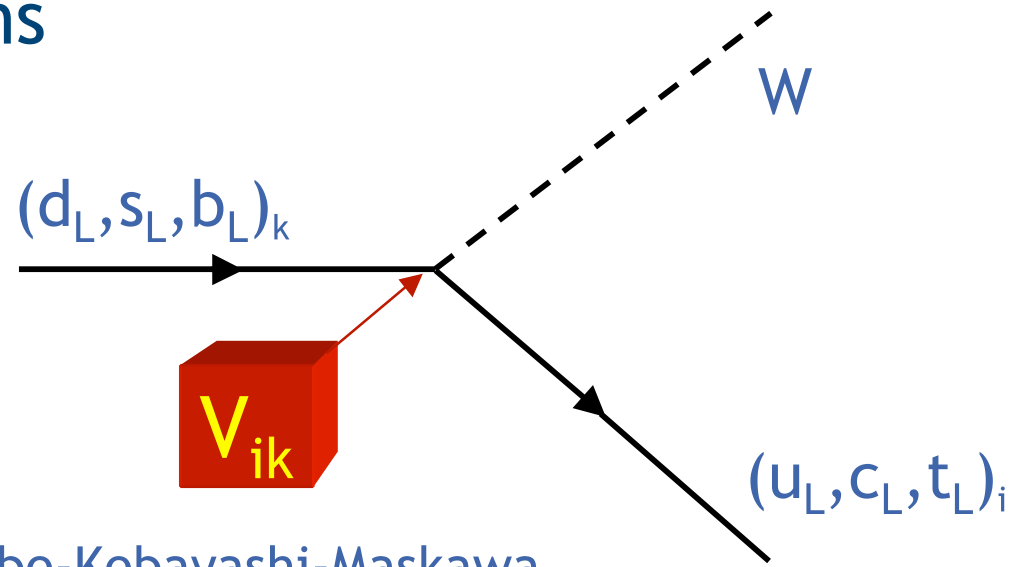
- Hierarchies in fermion mass spectrum:



- Likewise, hierarchies in quark mixings

Flavor physics

- Flavor physics studies communication between different generations
- Standard Model: present only in charged-current interactions



Cabibbo-Kobayashi-Maskawa matrix elements

Yukawa Couplings, CKM Matrix and Unitarity Triangle

Yukawa couplings

- Most general, gauge invariant and renormalizable interactions of Higgs and matter fields:

generation index

				SU(2)$_L$	U(1)$_Y$
L_L^i :	$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$,	$\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$,	$\begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$	2	-1/2
Q_L^i :	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$,	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$,	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	2	+1/6
e_R^i :	e_R ,	μ_R ,	τ_R	1	-1
u_R^i :	u_R ,	c_R ,	t_R	1	+2/3
d_R^i :	d_R ,	s_R ,	b_R	1	-1/3

Yukawa couplings

	$SU(2)_L$	$U(1)_Y$
$\Phi : \quad \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \end{pmatrix}, \qquad \widetilde{\Phi} = i\sigma_2\Phi^* : \quad \begin{pmatrix} \phi_2^{*0} \\ -\phi_1^{*-} \end{pmatrix}$	2	±1/2

- Yukawa couplings:

$$\boxed{\mathcal{L}_Y = -\bar{e}_R^i\, Y_e^{ij}\, \Phi^\dagger\, L_L^j - \bar{d}_R^i\, Y_d^{ij}\, \Phi^\dagger\, Q_L^j - \bar{u}_R^i\, Y_u^{ij}\, \widetilde{\Phi}^\dagger\, Q_L^j + \text{h.c.}}$$

Y: 1 -1/2 -1/2 1/3 -1/2 +1/6 -2/3 +1/2 +1/6

- Y_e, Y_d, Y_u: arbitrary complex 3x3 matrices
- Electroweak symmetry breaking: $\langle\phi_2^0\rangle = v/\sqrt{2}$

Yukawa couplings

- Gauge principle allows arbitrary generation-changing interactions, since fermions of different generations have equal gauge charges!
- Usually such couplings are eliminated by field redefinitions:

$$\psi^i \to U^{ij}\,\psi^j$$

unitary (i.e., probability preserving) “rotation” in generation space

Yukawa couplings

- Diagonalize Yukawa matrices using biunitary transformations, e.g.:

$$Y_e = W_e\,\lambda_e\,U_e^\dagger\,; \qquad \lambda_e = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

- Then perform field redefinitions:

$$e_L \to U_e\, e_L\,, \quad e_R \to W_e\, e_R$$
$$u_L \to U_u\, u_L\,, \quad u_R \to W_u\, u_R$$
$$d_L \to U_d\, d_L\,, \quad d_R \to W_d\, d_R$$

- This diagonalizes the mass terms, giving masses $m_f = y_f\,(v/\sqrt{2})$ to all fermions

CKM matrix

- Effect of field redefinitions on weak interactions in the mass basis (QCD and QED invariant)
- Charged currents:

$$\mathcal{L}_{\rm cc} = \frac{g_2}{\sqrt{2}} W^\mu \left(\bar u_L, \bar c_L, \bar t_L\right) \gamma_\mu V \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}\,; \qquad V = U_u^\dagger U_d$$

  – generation changing couplings proportional to V_{ij}:

$$d_L^{\,i} \to u_L^{\,j} + W^- \;\propto V_{ji} \qquad\qquad u_L^{\,i} \to d_L^{\,j} + W^+ \;\propto V_{ij}^*$$

(Cabibbo-Kobayashi-Maskawa matrix)

CKM matrix

- Neutral currents:

$$\mathcal{L}_{\rm nc} = \frac{g_2}{\cos\theta_W}\, Z^\mu \sum_f \left[\bar f_L U_f^\dagger \left(T_f^3\, \frac{1-\gamma_5}{2} - Q_f\, \sin^2\theta_W \right) U_f f_L \right.$$

$$\left. + \bar f_R W_f^\dagger \left(-Q_f\, \sin^2\theta_W \right) W_f f_R \right]$$

cancel each other

  - no generation-changing interactions!
    (at level of elementary vertices)
  - GIM mechanism (Glashow-Iliopoulos-Maiani, 1970)
  - led to prediction of charm quark (K-$\bar{\text{K}}$ mixing)

CKM matrix

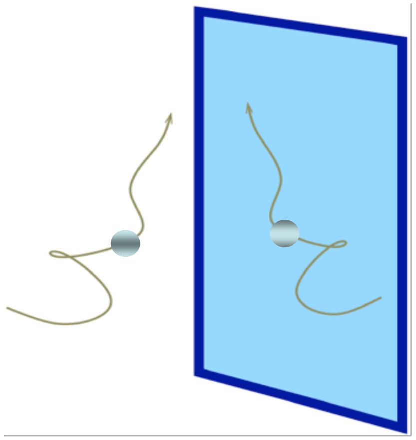
- Unitary 3x3 matrix V can by parameterized by 3 Euler angles und 6 phases
- Not all phases are observable, since under phase redefinitions $q \to e^{i\varphi_q} q$ of the quark fields:

$$V \to \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix}, \qquad V_{ij} \to e^{i(\varphi_d^i - \varphi_u^j)} V_{ij}$$

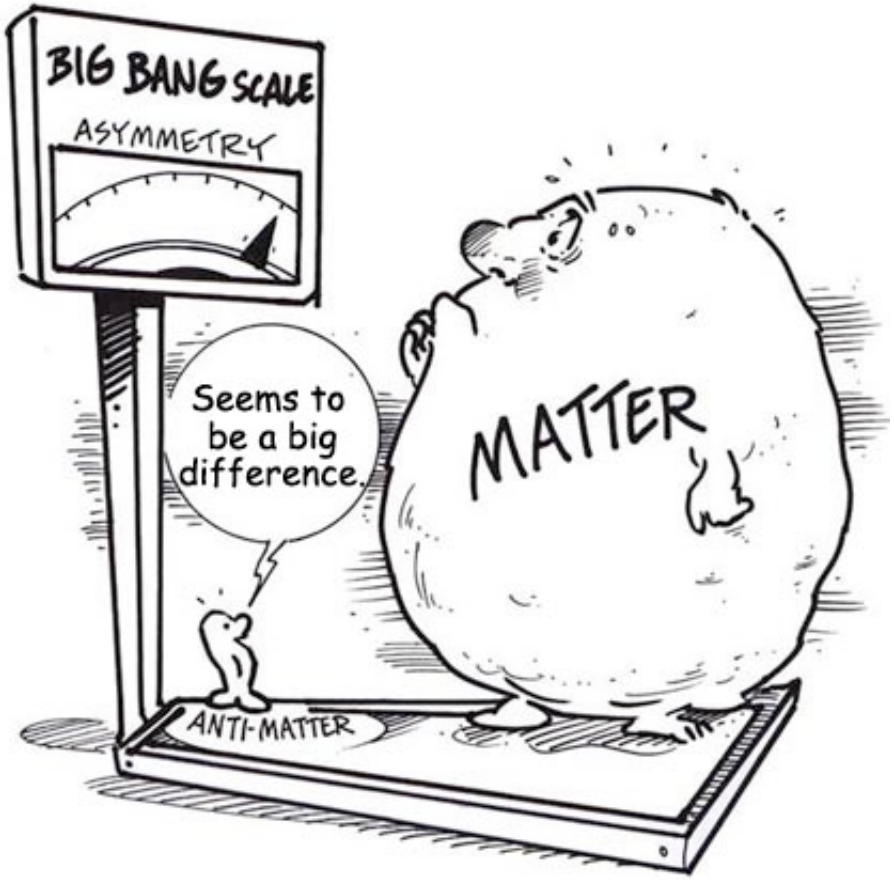
- 5 of 6 phases can be eliminated by suitable choices of phase differences!

CKM matrix

- Remaining phase δ_{CKM} is source of all CP-violating effects in Standard Model (assuming θ_{QCD}=0)
  - weak interactions couple to left-handed fermions and right-handed antifermions
  - violate P and C maximally, but would be invariant under CP and T if all weak couplings were real
  - physical phase of CKM matrix breaks CP invariance



- Allows for an absolute distinction between matter and antimatter!


BIG BANG SCALE
ASYMMETRY
Seems to be a big difference.
MATTER
ANTI-MATTER

CKM matrix

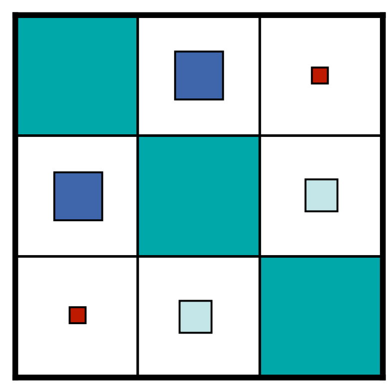
- CP violation required to explain the different abundances of matter and antimatter in the universe (baryogenesis)
- CP violation in quark sector requires N≥3 fermion generations
- Model for explanation of CP violation led to prediction of the third generation!
  Kobayashi, Maskawa (1973)

CKM matrix

- Form of V not unique (phase conventions)
- Several parameterizations used; a very useful one is due to Wolfenstein (1983):

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- Hierarchical structure in $\lambda \approx 0.22$
- Remaining parameters O(1)
- Complex entries $O(\lambda^3)$



CKM matrix

- Jarlskog determinant:
  for arbitrary choice of i,j,k,l the quantity

$$\boxed{\mathrm{Im}(V_{ij}V_{kl}V_{il}^{*}V_{kj}^{*}) = J \sum_{m,n} \varepsilon_{ikm}\, \varepsilon_{jln}}$$

  is an invariant of the CKM matrix (independent of phase conventions)
- CP invariance is broken if and only if $J\neq 0$
- Wolfenstein parameterization:

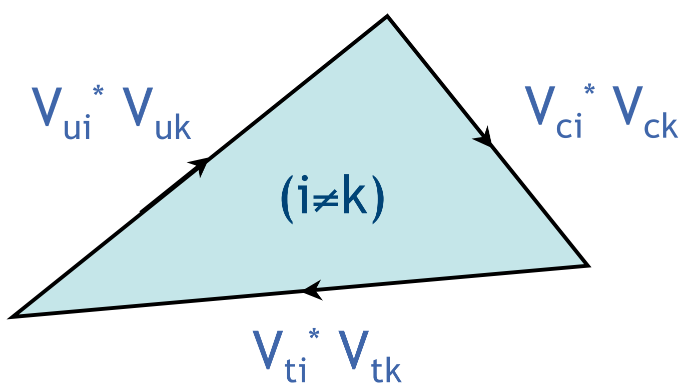
$$J = O(\lambda^6) = O(10^{-4}) \quad \text{rather small}$$

Unitarity triangle

- Unitarity relation $V^\dagger V = V V^\dagger = 1$ implies:

$$\boxed{V_{ji}^* V_{jk} = \delta_{ik} \quad \text{and} \quad V_{ij}^* V_{kj} = \delta_{ik}}$$

- For i≠k this gives 6 triangle relations, in which a sum of 3 complex numbers adds up to zero:



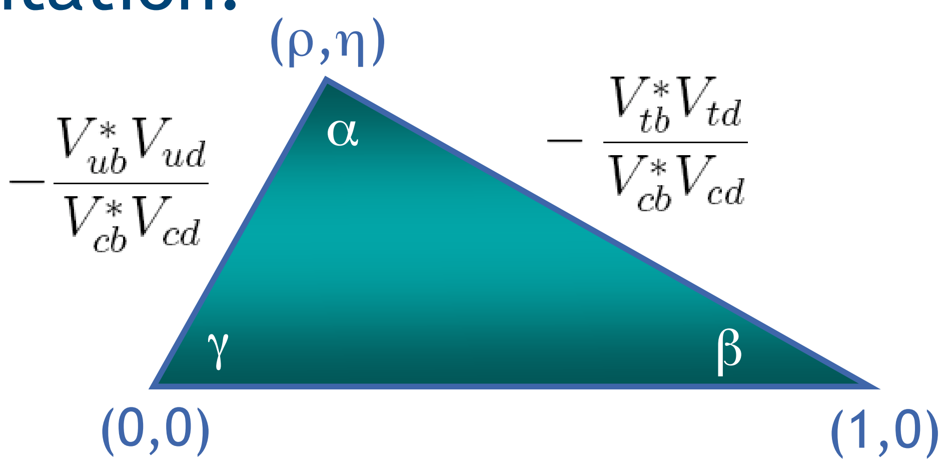
area = J/2

Unitarity triangle

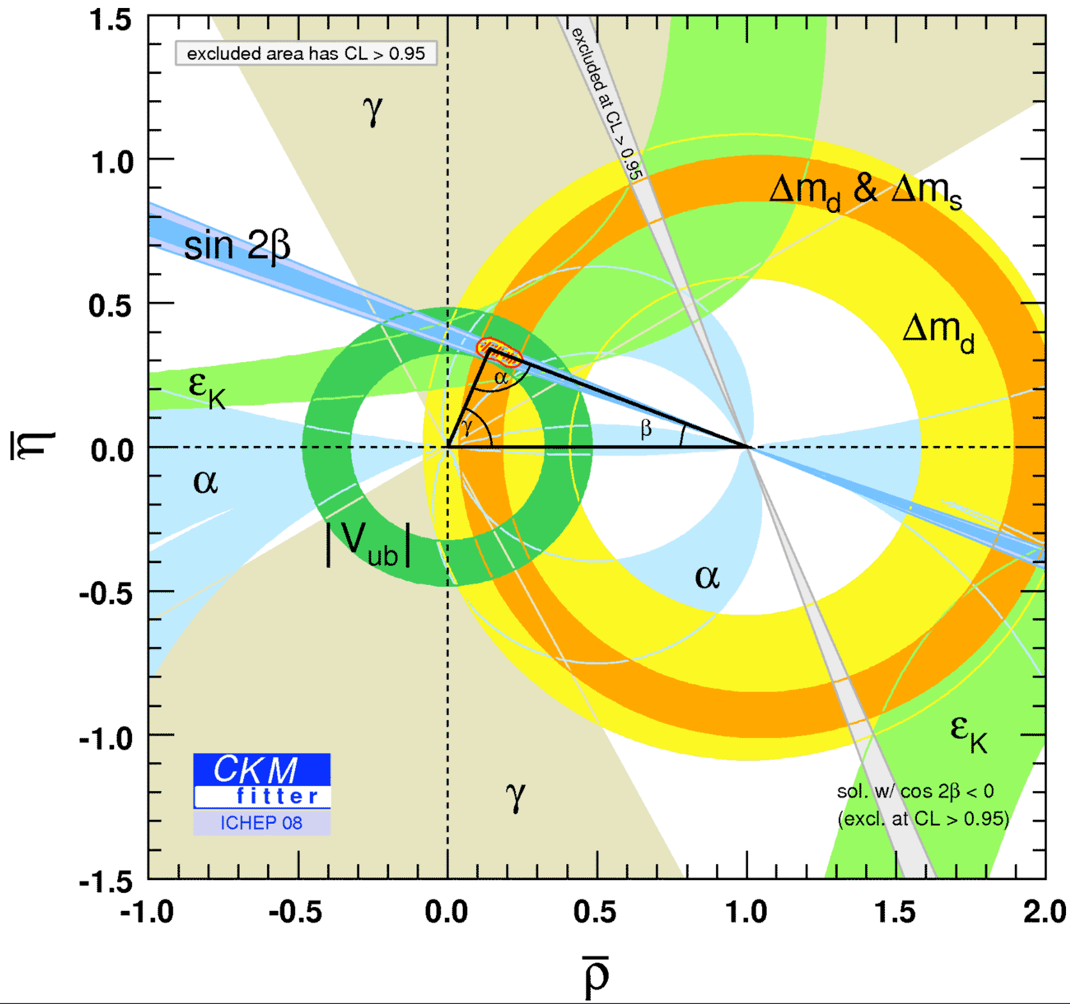
- Phase redefinitions turn triangles
- For two triangles, all sides are of same order in λ; *the* unitarity triangle is:

$$\boxed{V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0}$$

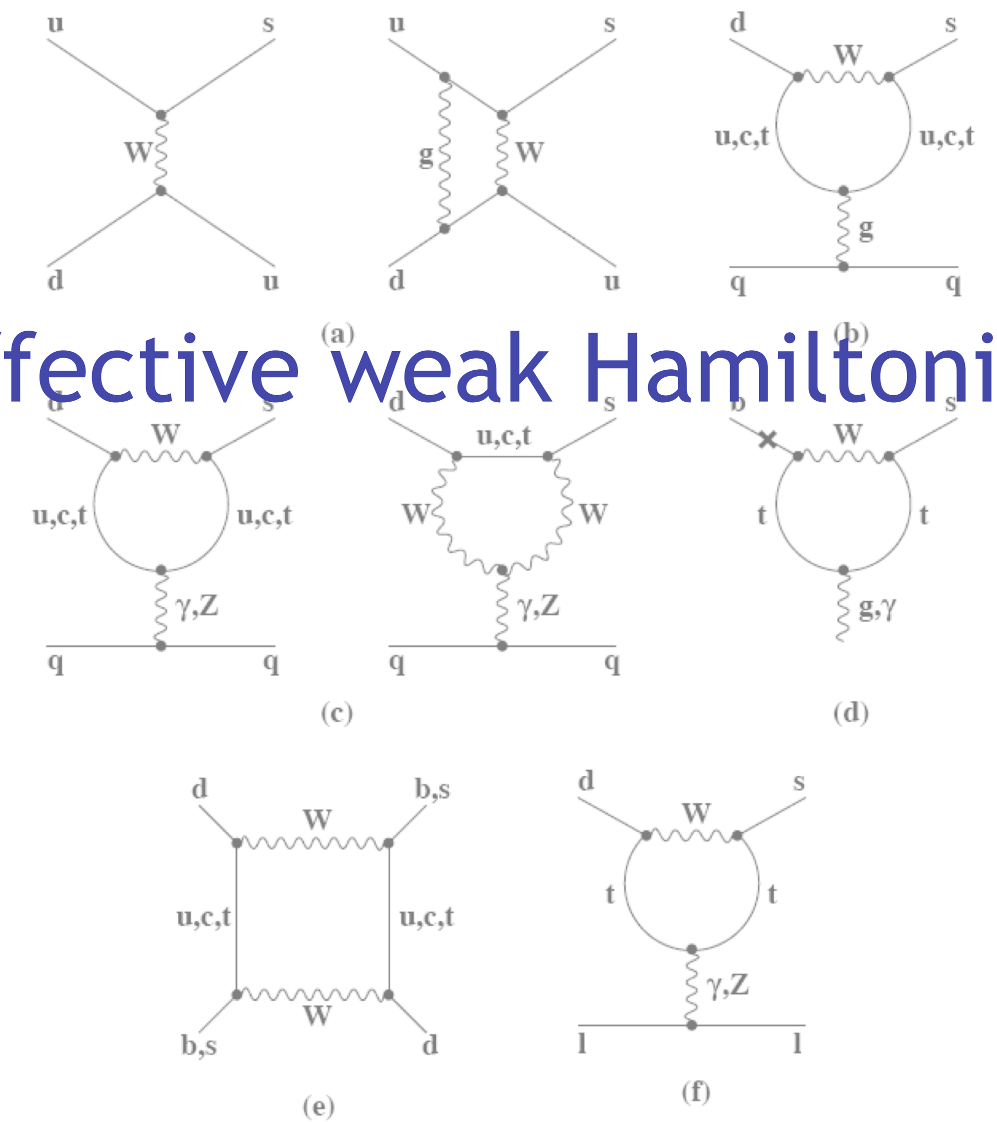
- Graphical representation:



Unitarity triangle determinations



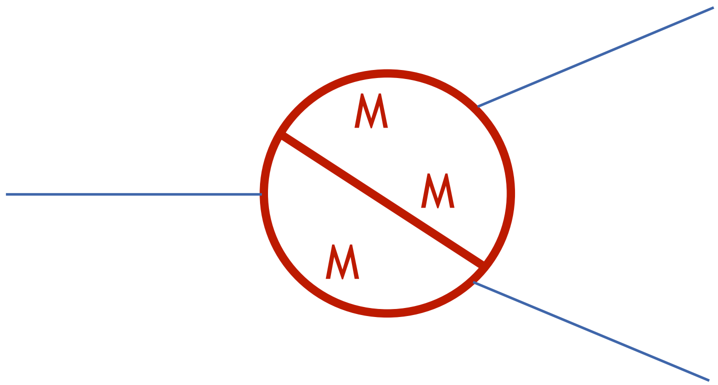
Effective weak Hamiltonian



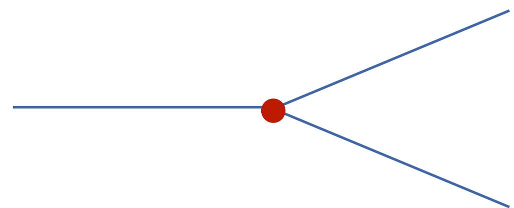
(a) (b) (c) (d) (e) (f)

Effective field theory

- At low energies, the exchange of heavy, virtual particles (M»E) leads to local effective interactions



exchange of heavy, virtual particles
between light SM particles



induced, effective local interactions
at low energies

- Effective field theory offers systematic description of effects of modes with large virtualities through an expansion in local operators

Effective field theory

- Standard Model is most successful effective field theory to date, even though it leaves open some questions:

Higgs mass (hierarchy problem)

cosmological constant

renormalizable quantum field theories

$$\mathcal{L}_{\rm EFT} = c^{(0)}\, M^4 + c^{(2)}\, M^2\, O^{(d=2)} + \sum_i c_i^{(4)}\, O_i^{(d=4)}$$

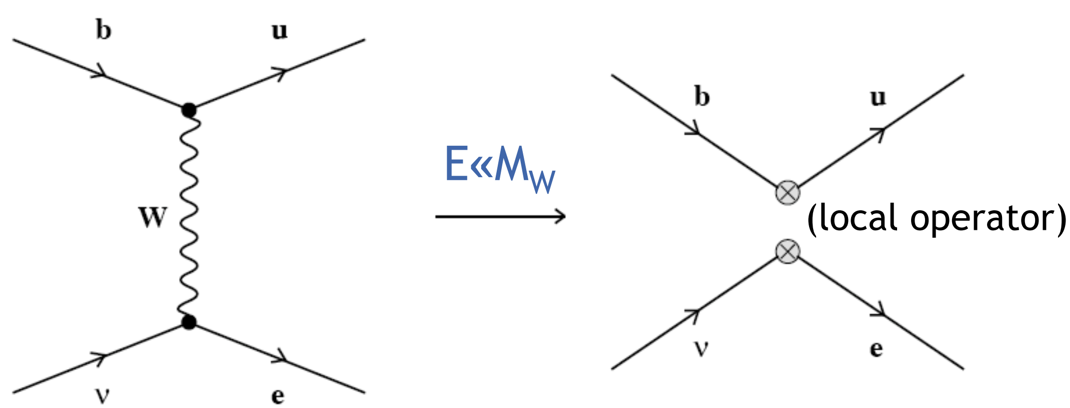
$$+ \frac{1}{M} \sum_i c_i^{(5)}\, O_i^{(d=5)} + \frac{1}{M^2} \sum_i c_i^{(6)}\, O_i^{(d=6)} + \dots$$

neutrino masses (see-saw mechanism)

possible effects of "new physics", proton decay, flavor physics, …

W exchange at low energies

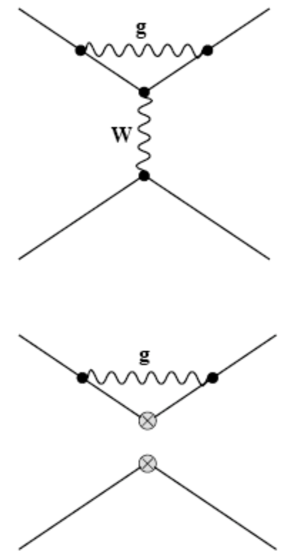
- Fermi theory of weak interactions describes W-boson exchange in terms of local 4-fermion couplings
- Consider:



- Fermi constant: $G_F/\sqrt{2} = g_2^2/8M_W^2$
  - determines scale of weak interactions

W exchange at low energies

- Semileptonic decay: QCD corrections influence both graphs in same way
- Resulting "effective" interaction for $E \ll M_W$:



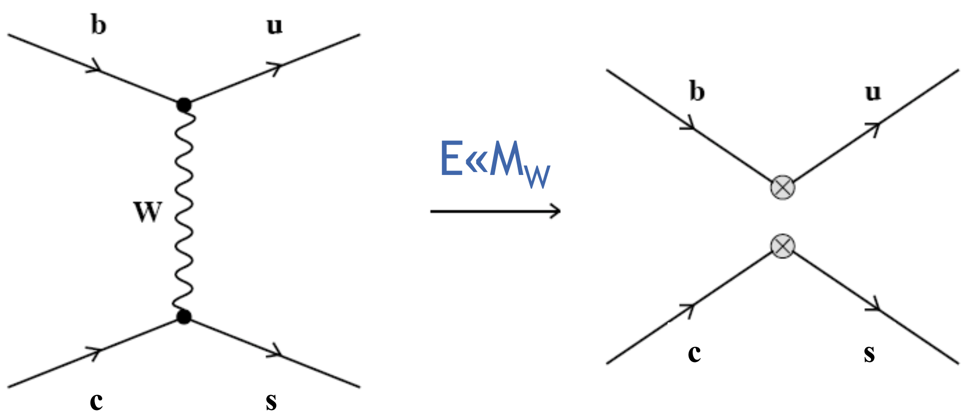
$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\, V_{ub}\, C_1(\mu)\, \bar{e}_L \gamma_\mu \nu_L\, \bar{u}_L \gamma^\mu b_L$$

$C_1=1$

- Scaling $1/M_W^2$ for d=6 operators explains weakness of "weak" interactions

W exchange at low energies

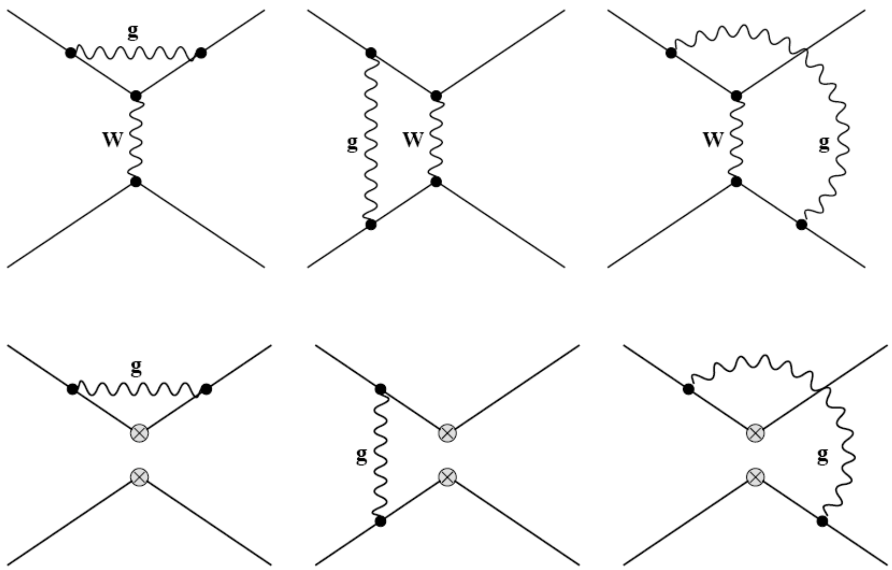
- W exchange between four different quark fields (nonleptonic decays):



- At tree level, analogous treatment as before

W exchange at low energies

- Complications for loop graphs:

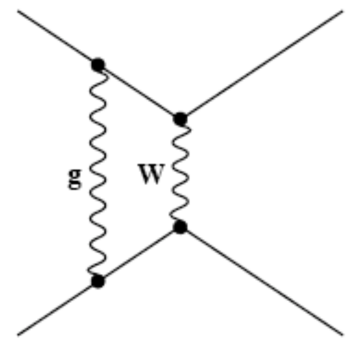


- Naïve Taylor expansion of W-boson propagator no longer justified!

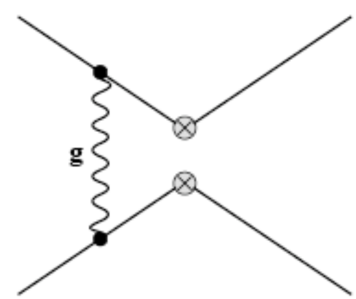
W exchange at low energies

- Problem with large loop momenta:

$$\int d^Dp \, \frac{1}{M_W^2 - p^2} \, f(p) \neq \frac{1}{M_W^2} \int d^Dp \left(1 + \frac{p^2}{M_W^2} + \dots \right) f(p)$$



- But no differences at low loop momenta!
- Effect can be calculated and corrected for using perturbation theory, since effective coupling $\alpha_s(M_W)$ is small



W exchange at low energies

- Resulting effective interaction:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\, V_{cs}^*\, V_{ub}\, \big[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j\, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j\, \bar{u}_L^j \gamma^\mu b_L^i\big]$$

with Wilson coefficients:

$$C_1(\mu) = 1 + \frac{3}{N_c}\,\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

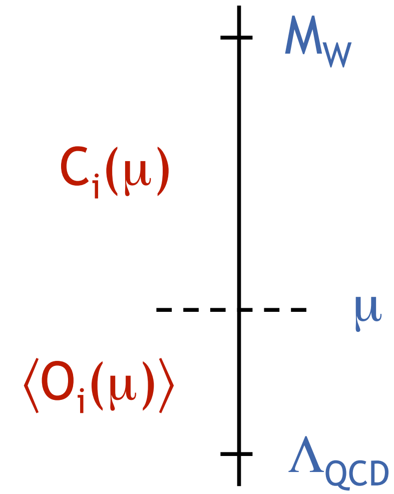
$$C_2(\mu) = -3\,\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,.$$

$\rightarrow$ accounts for effects of hard gluons (p~M_W)

Idea of effective field theory

- Separation of short- and long-distance effects; schematically:

$$\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}$$



- Short-distance effects ($p \sim M_W$) are perturbatively calculable
- Long-distance effects must be treated using nonperturbative methods
- Dependence on arbitrary separation scale μ controlled by RG equations

Idea of effective field theory

- Why useful?
- Any sensitivity to high scales (including to physics beyond the Standard Model) can be treated using perturbative methods:

$$\boxed{C_i(\mu) = C_i^{SM}(M_W, m_t, \mu) + C_i^{NP}(M_{NP}, g_{NP}, \mu)}$$

- Nonperturbative methods (operator product expansion, lattice gauge theory, …) usually only work at low scales (typically μ~few GeV)

FCNC processes

- While generation-changing couplings of W bosons to quarks exist, flavor-changing neutral currents such as

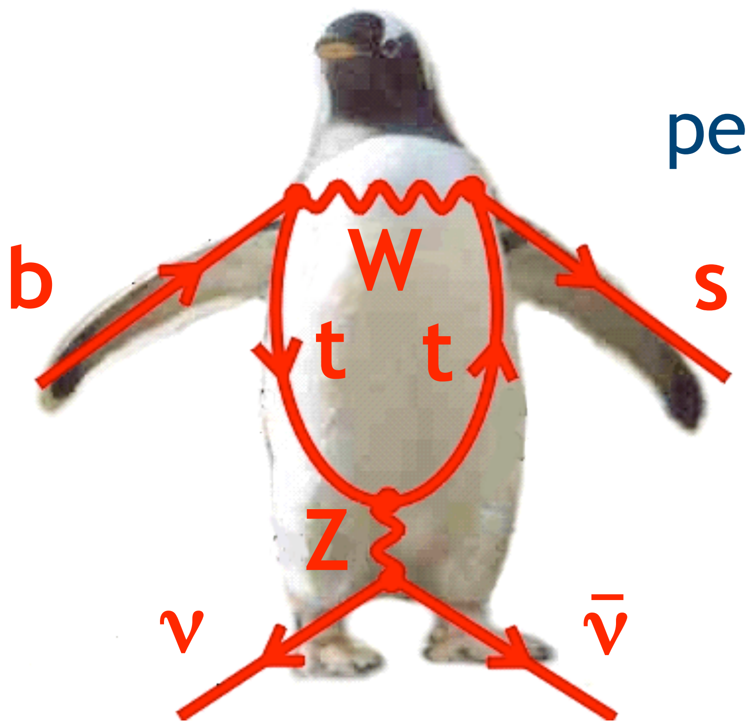
$$b\to s\gamma,\ \ b\to sZ^0,\ \ b\to s\nu\bar{\nu},\ \ b\to sd\bar{d},\ \ b\bar{d}\to d\bar{b},\ \ \text{etc.}$$

(and others, also for light quarks)

do not exist as elementary vertices in the Standard Model (GIM mechanism)

FCNC processes

- But such processes can be induced at loop level, e.g.:

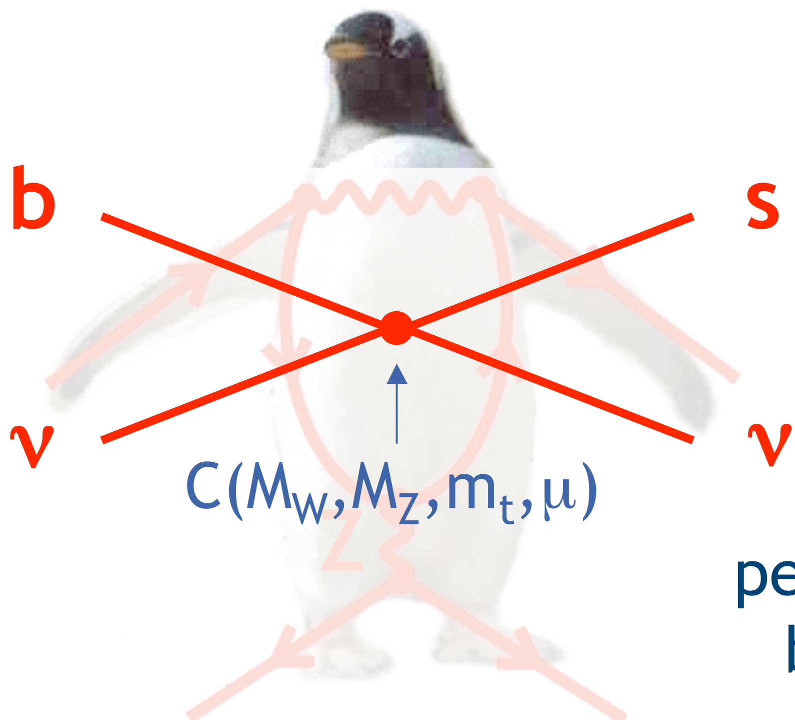


penguin diagram

loop-induced decay $b \to s\nu\bar{\nu}$

FCNC processes

- Effective interaction at low energies ($E \ll M_W, M_Z, m_t$):



penguin diagram approximated by local 4-fermion operator

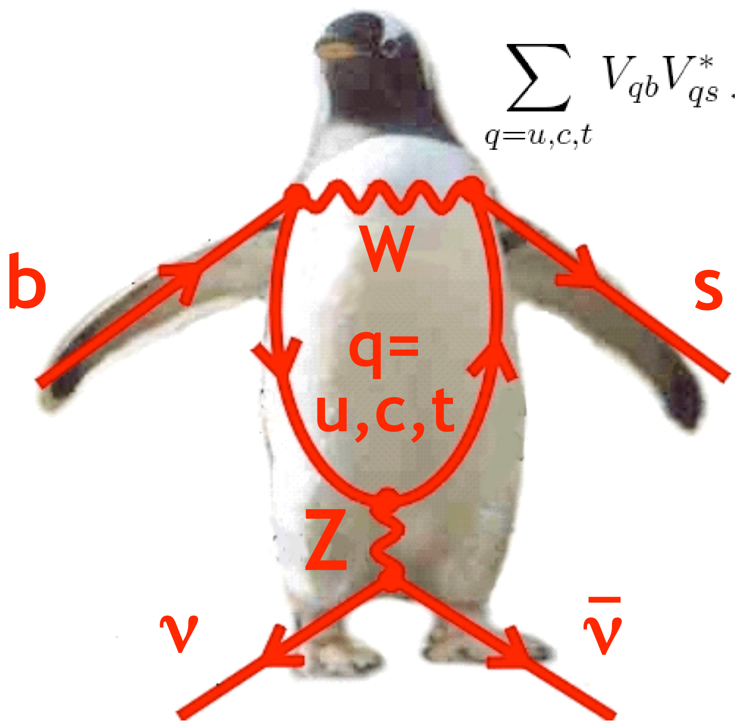
FCNC processes

- Detailed analysis (penguin autopsy) exhibits that GIM mechanism is “incomplete” in this case:

How to kill a penguin …

FCNC processes

- Detailed analysis (penguin autopsy) exhibits that GIM mechanism is “incomplete” in this case:



$$\sum_{q=u,c,t} V_{qb} V_{qs}^* \, f\Big(\frac{m_q^2}{M_W^2},\dots\Big) = V_{tb} V_{ts}^* \left[f\Big(\frac{m_t^2}{M_W^2},\dots\Big) - f\Big(\frac{m_u^2}{M_W^2},\dots\Big) \right]$$

$$+ V_{cb} V_{cs}^* \left[f\Big(\frac{m_c^2}{M_W^2},\dots\Big) - f\Big(\frac{m_u^2}{M_W^2},\dots\Big) \right]$$

Unitarity relation:

$$V_{tb} V_{ts}^* + V_{cb} V_{cs}^* + V_{ub} V_{us}^* = 0$$

$\rightarrow$ residual effect due to nontrivial mass dependence, often $\propto (m_t/M_W)^2$ or $\ln(m_t/\mu)$

FCNC processes

- Rich structure of couplings of Z^0,g,γ lead to a plethora of effective local d=6 operators
- Consider, e.g., decays of type b→s+X (or b→d+X, s→d+X), where X is flavor neutral:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} V_{qb} V_{qs}^* \left(C_1 \, Q_1^{(q)} + C_2 \, Q_2^{(q)} \right) - V_{tb} V_{ts}^* \sum_{i=3,\dots,10,7\gamma,8g} C_i \, Q_i \right]$$

W-boson exchange

penguin and box graphs

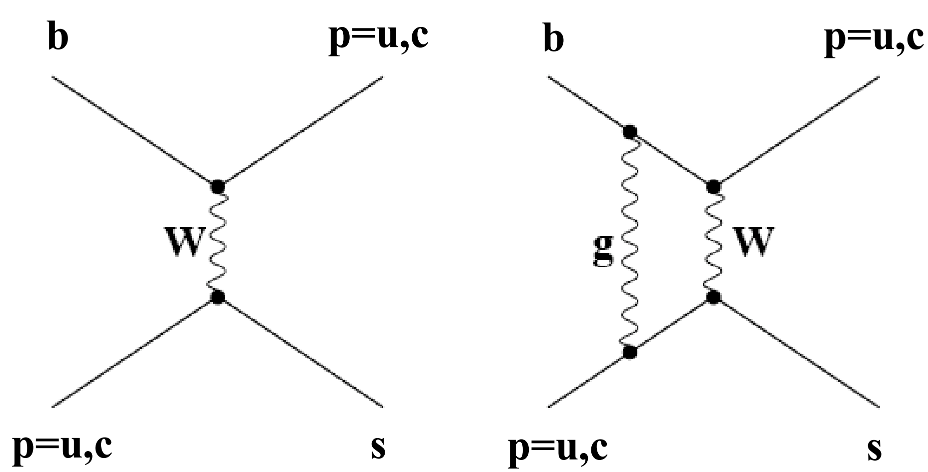
Operator basis

- Current-current operators (W exchange):

$$Q_1^{(p)} = (\bar{s}_i p_i)_{V-A}\,(\bar{p}_j b_j)_{V-A}$$

$$Q_2^{(p)} = (\bar{s}_i p_j)_{V-A}\,(\bar{p}_j b_i)_{V-A}$$

$$\boxed{(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2}$$



- Results analogous to earlier discussion):

$$C_1(M_W) = 1 - \frac{11}{6}\,\frac{\alpha_s(M_W)}{4\pi}$$

$$C_2(M_W) = \frac{11}{2}\,\frac{\alpha_s(M_W)}{4\pi}\,,$$

← results quoted at $\mu = M_W$ for simplicity

Operator basis

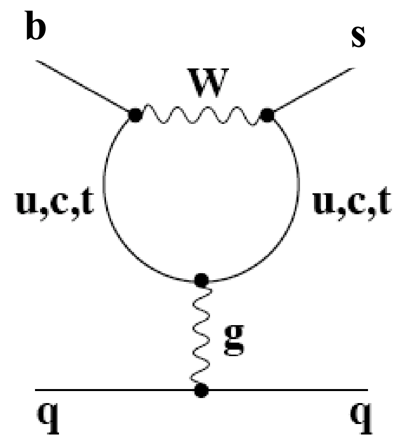
- QCD penguin operators:

$$Q_3 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_j)_{V-A}$$

$$Q_4 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_i)_{V-A}$$

$$Q_5 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_j)_{V+A}$$

$$Q_6 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_j q_i)_{V+A}$$



- Results:

$$C_3(M_W) = C_5(M_W) = -\frac{1}{6}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}$$

$$C_4(M_W) = C_6(M_W) = \frac{1}{2}\,\widetilde{E}_0\left(\frac{m_t^2}{M_W^2}\right)\frac{\alpha_s(M_W)}{4\pi}$$

Loop function:

$$\widetilde{E}_0(x) = -\frac{7}{12} + O(1/x)$$

Operator basis

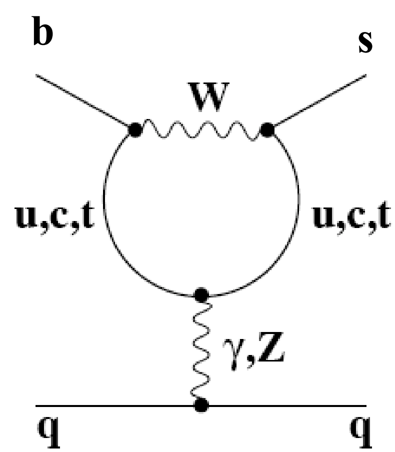
- Electroweak penguin operators:

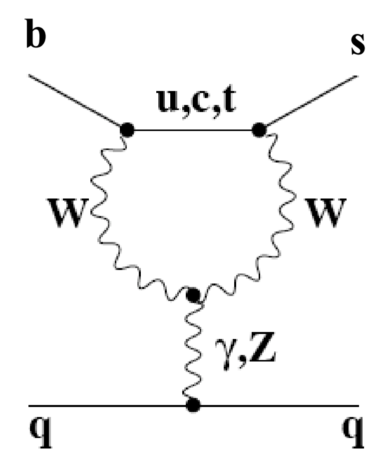
$$Q_7 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_j)_{V+A}$$

$$Q_8 = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_i)_{V+A}$$

$$Q_9 = (\bar{s}_i b_i)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_j)_{V-A}$$

$$Q_{10} = (\bar{s}_i b_j)_{V-A} \sum_{q=u,d,s,c,b} \frac{3}{2} e_q \, (\bar{q}_j q_i)_{V-A}$$





- Results:

$$C_7(M_W) = f\left(\frac{m_t^2}{M_W^2}\right) \frac{\alpha(M_W)}{6\pi}, \qquad C_8(M_W) = C_{10}(M_W) = 0$$

$$C_9(M_W) = \left[f\left(\frac{m_t^2}{M_W^2}\right) + \frac{1}{\sin^2\theta_W}\, g\left(\frac{m_t^2}{M_W^2}\right)\right] \frac{\alpha(M_W)}{4\pi}$$

Loop functions:

$$f(x) = \frac{x}{2} + \frac{4}{3}\ln x - \frac{125}{36} + O(1/x)$$

$$g(x) = -\frac{x}{2} - \frac{3}{2}\ln x + O(1/x)$$

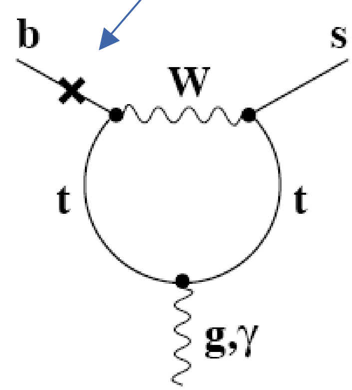
Operator basis

- Dipol operators:

$$Q_{7\gamma} = -\frac{e m_b}{8\pi^2}\, \bar{s}\, \sigma_{\mu\nu}\, (1+\gamma_5)\, F^{\mu\nu}\, b$$

$$Q_{8g} = -\frac{g_s m_b}{8\pi^2}\, \bar{s}\, \sigma_{\mu\nu}\, (1+\gamma_5)\, G_a^{\mu\nu} t_a\, b$$

chirality flip $\propto m_b$



- Results ($x=m_t^2/M_W^2$) :

$$C_{7\gamma}(M_W) = -\frac{1}{3} + O(1/x)$$

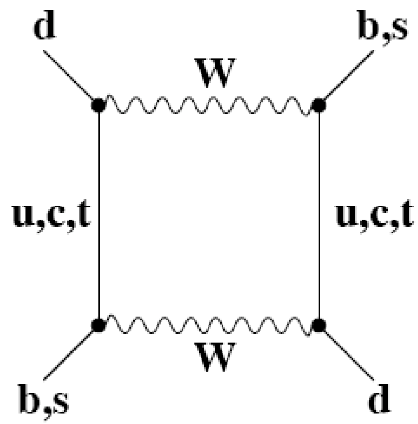
$$C_{8g}(M_W) = -\frac{1}{8} + O(1/x)$$

That's it !
(apart from operators containing leptons …)

FCNC processes

- Consider finally B-$\bar{\text{B}}$ mixing processes mediated by transitions $b\bar{d}\to d\bar{b}$ or $b\bar{s}\to s\bar{b}$
- Effective interaction:

$$\mathcal{H}_{\rm eff} \propto G_F^2 M_W^2 (V_{tb}V_{td}^*)^2\, S_0\!\left(\frac{m_t^2}{M_W^2}\right) (\bar{d}b)_{V-A}(\bar{d}b)_{V-A}$$



  - dominant contribution by far ($\propto m_t^2$) due to top-quark loop
  - first hint toward very heavy top quark